الدوال العكسية

- properties of functions

① $y = 2x + 5 \longrightarrow$ Relation
$x^2 + y^2 = 4 \longrightarrow$ function

(each vertical line intersect with function in one point)
(each value of x have one value of y)

→ Is every function have inverse?
x is function of y (when each value of x has one and it is converse value of y).

② ex $y = x^2$ (not inv.)
$y = 2x + 5$ (inv.).
(each horizontal line intersect curve in one point).

→ one to one function. (y is function of x) (the only function that has inverse). (x is function of y) } The condition one to one

if $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$
$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ } definition

⇒ From (1) $2x_1 + 5 = 2x_2 + 5$ $\therefore x_1 = x_2$ (one to one func)
⇒ From (2) $x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$ $\therefore x_1 \neq x_2$ (not one to one

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$y = x^2$ $(x \geq 0) \Rightarrow$ is one to one when there is a condition in specific interval.

→ The relation between function and inverse?
$y = f(x) \rightarrow$ function
$x = f^{-1}(y) \longrightarrow$ The Form of inverse function
from (1) $f(x) = 2x + 5$ $x = f^{-1}(y) = \frac{y-5}{2}$
$f^{-1}(x) = \frac{x-5}{2}$

- The Relation between function and inverse : (f and f^{-1})
  - Domain of f = Range of f^{-1}
  - Range of f = domain of f^{-1}.
  - graph is symmetric about ($y=x$)

$f^{-1}(f(x)) = x$, $f(f^{-1}(x)) = x$

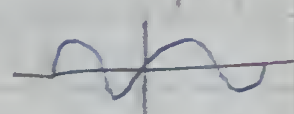
- inverse function : is a function of (one to one function)
  - ➡ if a function isn't one to one (There is an inverse function of specific interval).

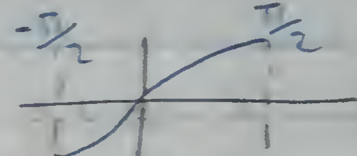
· Inversed Triagometric functions :

$y = \sin x$ ⟹ inverse : $y = \sin^{-1} x \neq \frac{1}{\sin x} = (\sin x)^{-1}$

Domain : R ⟹ $[-\frac{\pi}{2}, \frac{\pi}{2}]$ to be one to one function.

Range : $[-1, 1]$

Graph : 
(not one to one)

Graph : 

③ $y = \sin^{-1} x$ ⟹ D : $[-1, 1]$, Range $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

→ Graph : 

→ Drevetive : $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

⟹ From ③

$\sin y = \sin(\sin^{-1} x) = x$

$x = \sin y$

$\frac{dx}{dy} = \cos y = \sqrt{1-\sin^2 y}$

$= \sqrt{1-x^2}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

Drevetive : $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

NOTE

$\boxed{\frac{dy}{dx} = \left(\frac{1}{dx/dy}\right)}$

$y = 2x+1$

$\frac{dy}{dx} = 2$

$x = \frac{y-1}{2}$

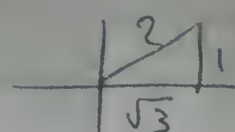
$\frac{dx}{dy} = \frac{1}{2}$

(complete drevetive)

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$, if $\Rightarrow$ $\frac{d}{dx}\left(\sin^{-1} u(x)\right) = \frac{u'(x)}{\sqrt{1-u^2(x)}}$

$\rightarrow$ $u'(x) \rightarrow$ Derevetive of function but (u not u^2)

$\sin x = \frac{1}{2}$

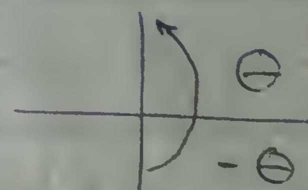


$\sin^{-1} \frac{1}{2} = \frac{\pi}{6}$

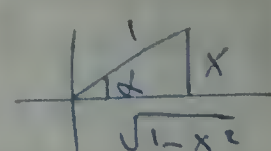
as Range of $\sin^{-1} x$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\sin^{-1}\left(\frac{-1}{2}\right) = -\frac{\pi}{6}$

<u>Note</u>: each angle in the fourth quarter is (-ve).
as Domain Range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. not $[0, 2\pi)$.



$\cos(\sin^{-1} x) \Rightarrow \sin^{-1} x = \alpha$
$x = \sin\alpha.$



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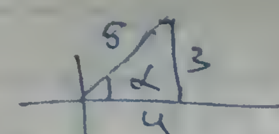
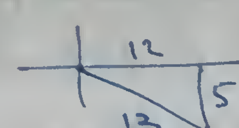
ex
$\Rightarrow \tan\left(\underbrace{\sin^{-1}\frac{3}{5}}_{\alpha} + \underbrace{\sin^{-1}\left(-\frac{5}{13}\right)}_{\beta}\right)$

1) $\sin^{-1}\frac{3}{5} = \alpha$ $\sin\alpha = \frac{3}{5}$



$\tan\alpha = \left(\frac{3}{4}\right)$

2) $\sin^{-1}\left(-\frac{5}{13}\right) = \beta$ $\sin\beta = \frac{-5}{13}$



$\tan\beta = \left(\frac{-5}{12}\right)$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \frac{\left(\frac{3}{4}\right)+\left(\frac{-5}{12}\right)}{1-\left(\frac{3}{4}\right)\left(\frac{-5}{12}\right)}$.

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ex
$\Rightarrow$ $y = \sin^{-1}(x^2)$

$y' = \frac{2x}{\sqrt{1-(x^2)^2}}$

$(x^2)^2 \Rightarrow$ The function squared
$(2x) \Rightarrow$ derevitive of function not function's squared

ex $\Rightarrow$ $y = \sin^{-1}(e^{2x})$.

$y' = \frac{2e^{2x}}{\sqrt{1-(e^{2x})^2}}$

ex $\Rightarrow$ $y = \sin^{-1}(\sqrt{x^2+1})$

$y' = \frac{\frac{2x}{2\sqrt{x^2+1}}}{\sqrt{1-(x^2+1)}}$

$\underline{ex} \longrightarrow y = \sin^{-1}(\cos x).$

$$y' = \frac{-\sin x}{\sqrt{1-\cos^2 x}} =$$

$$= \frac{-\sin x}{\sin x} = \boxed{-1}$$

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